The vision of Bill Thurston

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William Thurston was a world-renowned mathematician. His ideas revolutionized many areas of geometry and topology[1]; the proof of his geometrization conjecture was eventually completed by Grigori Perelman, thus settling the Poincaré conjecture (making it the only solved Millennium Prize problem). After his death, his students wrote reminiscences, describing among other things his exceptional vision.[2] Here's Jeff Weeks:

> Bill's gift, of course, was his vision, both in the direct sense of seeing geometrical structures that nobody had seen before and in the extended sense of seeing new ways to understand things. While many excellent mathematicians might understand a complicated situation, Bill could look at the same complicated situation and find simplicity.

Thurston emphasized clear vision over algebra, even to a fault. Yair Minksy:

> Most inspiring was his insistence on understanding everything in as intuitive and immediate a way as possible. A clear mental image of a mathematical construction or proof was worth much more than a formalization or a calculation. […] If you were able to follow the images and the structure he was exploring, you obtained beautiful insights; but if you got lost, you were left with nothing […].

There was something else exceptional about Thurston's vision. Benson Farb:

> Bill was probably the best geometric thinker in the history of mathematics. Thus it came as a surprise when I found out that he had no stereoscopic vision, that is, no depth perception. Perhaps the latter was responsible somehow for the former? I once mentioned this theory to Bill. He disagreed with it, claiming that all of his skill arose from his decision, apparently as a first-grader, to "practice visualizing things" every day.

His vision problems were congenital (present from birth). Gabai and Kerckhoff:

> Bill had a congenital case of strabismus and could not focus on an object with both eyes, eliminating his depth perception. He had to work hard to reconstruct a three-dimensional image from two two-dimensional ones. Margaret worked with him for hours when he was two, looking at special books with colors. His love for patterns dates at least to this time. As a first-grader he made the decision "to practice visualization every day." Asked how he saw in four or five dimensions he said it is the same as in three dimensions: reconstruct things from two-dimensional projections.

With due reverence for the dead, one nevertheless wonders if his disorder did help stimulate him to use his self-authority to persistently practice visualization.

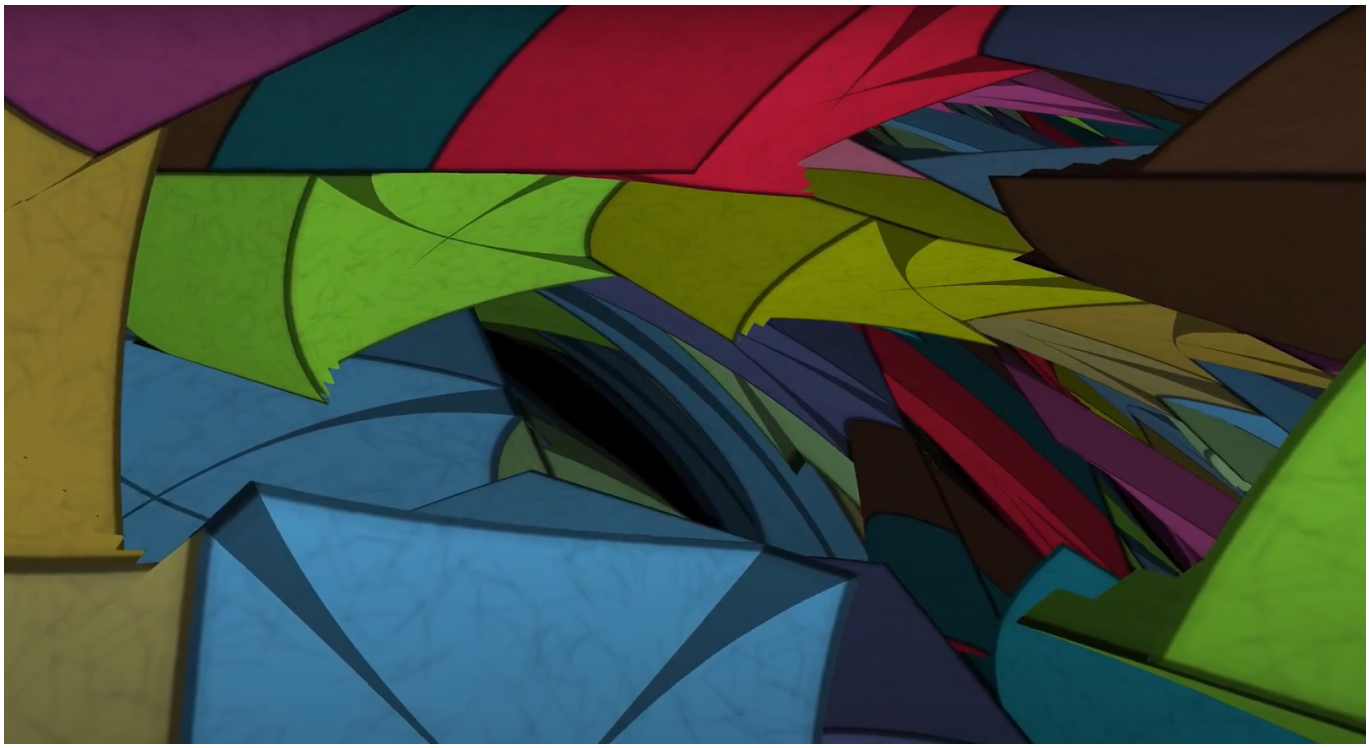
The genetics of strabismus aren't well-understood, but it's clear that in some cases it is genetic[3]. Now, strabismus is not so uncommon. Over 1% of children have the condition, and the great majority of such people don't become mathematical pioneers. Thus there's no open-and-shut case to say that Thurston's genetics necessarily much influenced his life's work in this way.

But it's at least plausible that he had a genetically-caused visual impairment that spurred him to develop uncommon ways of thinking that generalized powerfully and allowed him to see strange things that others did not see.

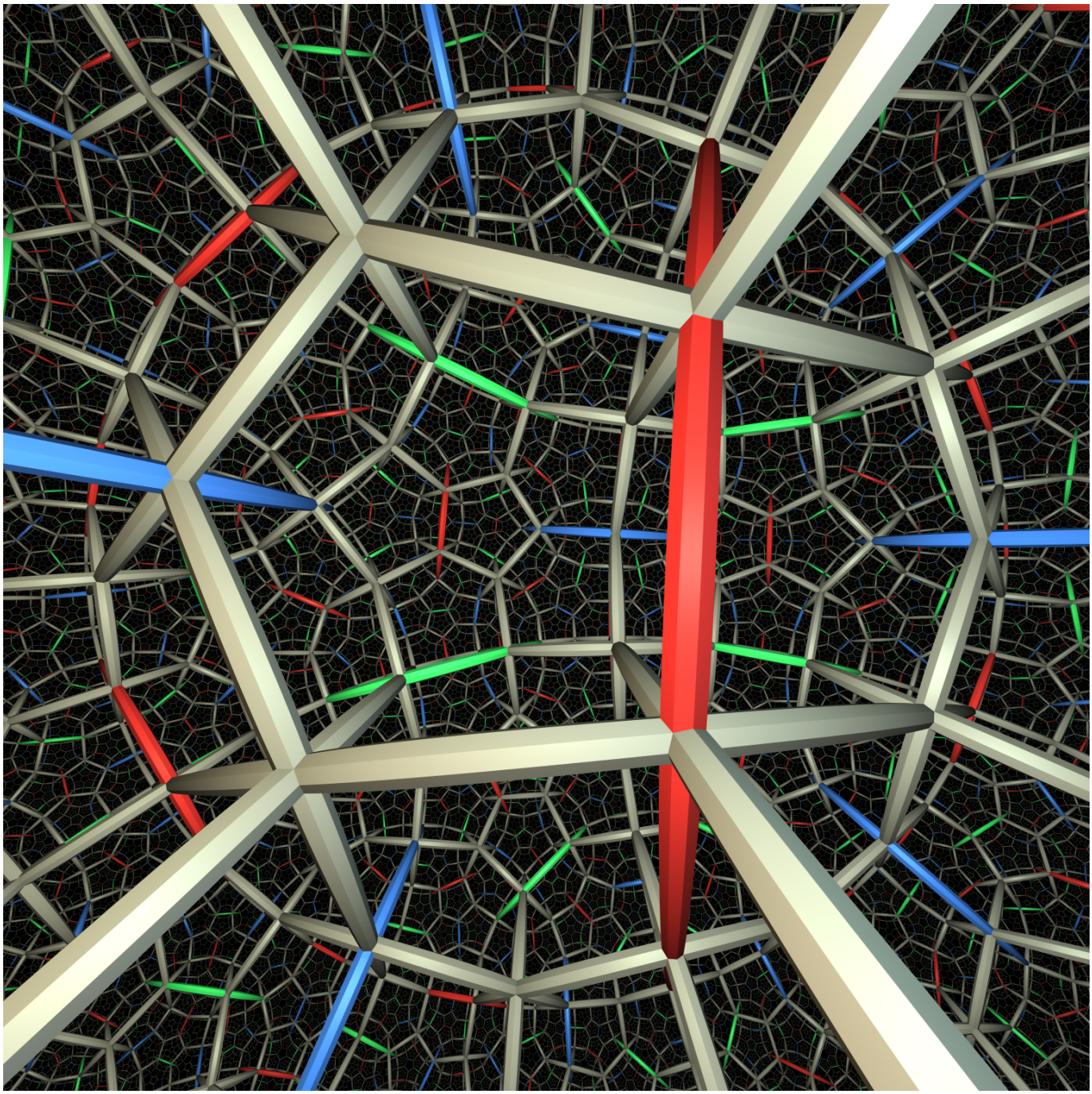
[1] PDF: https://www.ams.org/notices/201511/rnoti-p1318.pdf. Gabai, David, and Steve Kerckhoff. 'William P. Thurston, 1946–2012'. Notices of the American Mathematical Society 62, no. 11 (1 December 2015): 1318–32. https://doi.org/10.1090/noti1300.

[2] PDF: https://www.ams.org/publications/journals/notices/201601/rnoti-p31.pdf. Gabai, David, and Steve Kerckhoff. 'William P. Thurston, 1946–2012'. Notices of the American Mathematical Society 62, no. 11 (1 December 2015): 1318–32. https://doi.org/10.1090/noti1300.

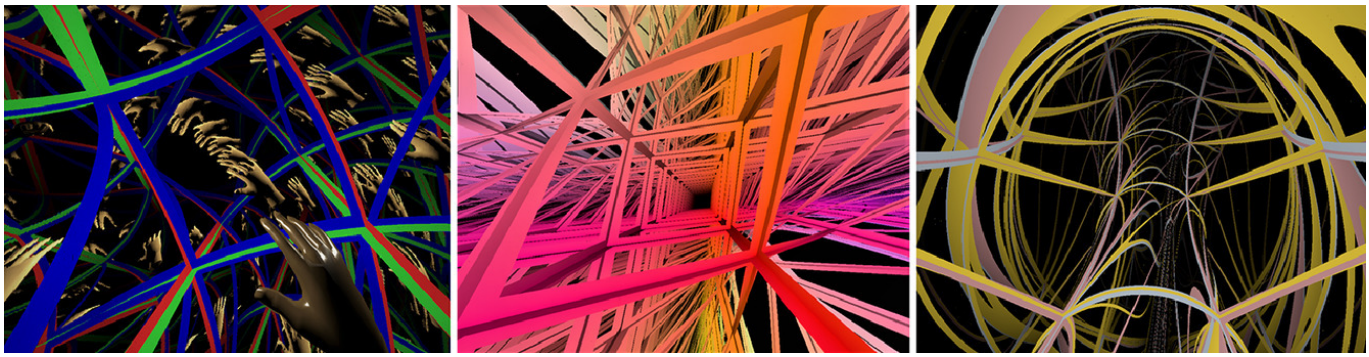
[3] https://www.frontiersin.org/journals/ophthalmology/articles/10.3389/fopht.2023.1233866/full. Martinez Sanchez, Mayra, and Mary C. Whitman. 'Genetics of Strabismus'. Frontiers in Ophthalmology 3 (20 July 2023). https://doi.org/10.3389/fopht.2023.1233866.



(Screenshot from "Nil Geometry Explained!" by ZenoRogue, 2022 (https://www.youtube.com/watch?v=FNX1rZotjjI), on one of the eight Thurston geometries. Thurston didn't discover these geometries, but formulated and advanced the conjecture that these eight geometries can be used to build up any closed 3-manifold (that is, any "finite" 3-dimensional space, maybe curvy and twisty with wormholes, as long as it doesn't have edges and it doesn't go on forever in some direction).)



(Visualization of hyperbolic 3-space, from Geometry Center[4].)



(From "Visualization of Nil, Sol, and SL2(R) Geometries"[5].)

The lesson I want to here draw from Thurston's vision is that even a condition which seems so obviously worse to have, is not at all obviously a *strict* detriment. If, arguendo, Thurston's special mathematical insight was in part provoked by his impairment, then that impairment had quite mixed consequences–a life of additional difficulty for Thurston, and a treasure trove for humanity's geometric thought.

My intent with this lesson is to give some pause to those who think that there's an unambiguous universal "good"

[4]'Geometry Center Videos, Revisited'. Accessed 27 March 2025. https://www.cs.ubc.ca/~tmm/gc/.

[5]Novello, Tiago, Vinícius da Silva, and Luiz Velho. 'Visualization of Nil, Sol, and SL2(R) Geometries'. Computers & Graphics 91 (1 October 2020): 219–31. https://doi.org/10.1016/j.cag.2020.07.016.

direction for genomes to go. That intuition leads people to suggest that it might be fine or good to prohibit or require certain uses of germline engineering. The assertion goes: If there is a disease, disability, or disorder, like blindness or deafness or dwarfism or autism, and if it's feasible to prevent that condition in a future child using germline engineering, then people should be required by law to do so if they can; or at least, it should be prohibited by law to *make* your child have such a condition. The condition harms the child, and there's no justification for propagating it.

Other people might view their own atypical condition as being of some special value, perhaps to them or to others, even if it comes with added difficulties. They may want to pass on to their child that special value, and to share in common with their child this way of being.

Thus there is a conflict.

My stance is: protection of genomic liberty for all. This stance does not imply that one *ought not* to prevent one's future child from having some sort of impairment, just because maybe hypothetically that could lead to the child gaining some unusual abilities in a few very rare cases. On the contrary, there's very good reason to choose to give one's child a genomic foundation for as much health and capability as possible[6]. Hopefully humanity can, with time, arrange things so that access to any unusual abilities or ways of being can be gained through the free choices made by, and the work exerted by, the children themselves, with no need for any exclusive genomic predisposition–as Thurston insisted was true of his own abilities.

But here I just want to suggestively ask, to those who say "No; disorder and decreased capability is obviously strictly worse and should be prevented through state intervention.": How sure are you that there aren't ways of being that you don't easily perceive or anticipate, but that are truly valuable to humanity at large and/or to the people who exist that way? Why impose your questionably-well-informed judgement on someone else's desire to propagate their way of being, without extremely good and near-undoubtable reason? If you come to a conclusion on this matter, it should be only after becoming able to appreciate the breadth of human ways of being.

A last quotation, from Benson Farb[7]:

> In interacting with other mathematical greats, one gets the feeling that these people are like us but just 100 (OK, 500) times better. In contrast, Thurston was a singular mind. He was an alien. There is no multiplicative factor here; Thurston was simply orthogonal to everyone. Mathematics loses a dimension with his death.

[6] Savulescu, Julian, and Guy Kahane. 'The Moral Obligation To Create Children With The Best Chance Of The Best Life'. Bioethics 23, no. 5 (June 2009): 274–90. https://doi.org/10.1111/j.1467-8519.2008.00687.x.

[7] PDF: https://www.ams.org/publications/journals/notices/201601/rnoti-p31.pdf. Gabai, David, and Steve Kerckhoff. 'William P. Thurston, 1946–2012'. Notices of the American Mathematical Society 62, no. 11 (1 December 2015): 1318–32. https://doi.org/10.1090/noti1300.